

UNCERTAINTY RELATION FOR QUANTUM GRAVITY *

Ya. I. AZIMOV

*Petersburg Nuclear Physics Institute,
National Research Center "Kurchatov Institute",
Gatchina, 188300, Russia
E-mail: azimov@thd.pnpi.spb.ru*

Discussion of physical realization of coordinates demonstrates that the quantum theory of gravity (still absent) should be non-local and, probably, non-commutative as well.

Various fields became familiar objects for contemporary physics. But their description may be different. Classical fields are usually described as numerical functions $A(x, y, z, t)$ of coordinates and time (and, m.b., of some other parameters). It is implicitly assumed that both the coordinates, time, and the field values can be fixed with arbitrary good precision. The above approach is applicable, in particular, to classical gravitation field.

Quantum fields are generally described as operator functions $A(x, y, z, t)$, also depending on coordinates and time. Usually, the field operator A may be diagonalized. And it is assumed again that both the coordinates and time, and the diagonal field values can be fixed with arbitrary good precision.

Quantum theory of gravitation is still absent, though the literature contains many models and attempts to construct it (see, *e.g.*, the recent reviews ^{1,2,3}). Many of their authors act usually in the familiar way and try to formulate the quantum gravity as a local field theory. Here we discuss, whether this is possible.

Having a particular coordinate frame, one may physically mark a particular physical point by means of a point-like test-body being at rest in the frame. By definition, the test-body should be chosen so to minimize

*Based on the talk at the 4th V.N.Gribov Memorial Workshop (gribov85), June, 17-20, 2015, Chernogolovka, Russia. To be published in the Proceedings.

its influence on the external fields. This can certainly be realized in the classical case or for the non-relativistic quantum case. In both cases, space coordinates can be fixed with arbitrary precision.

The situation looks different for the relativistic quantum case accounting for the finite velocity of light c . Position of a test-body of mass M , even if it is point-like, can not be fixed with better precision than its Compton length ⁴

$$\lambda_C = \frac{\hbar}{Mc} . \quad (1)$$

For most fields, this fact does not restrict precision of coordinates: for a better precision one should use a heavier test-body. However, if the gravitation field is switched on, one encounters the inconsistency: a heavier test-body provides a better precision for its position, but stronger distorts the external gravitation field.

The same conclusion may be obtained in other way(s). If consideration begins with gravitation field in non-relativistic case and/or without quantum effects, then one could take a test-body to have very small mass, to not perturb the external field. However, accounting for both relativistic and quantum effects in this situation generates large uncertainty in the position of such test-body.

Let us consider the situation in some more detail. To fix the space point (x, y, z) up to the sphere of the radius δ_r one should use a test-body with the mass

$$M_\delta \approx \frac{\hbar}{\delta_r c} . \quad (2)$$

Then, to the gravitation field φ near the point (x, y, z) , in the Newton approximation, the test-body appends the uncertainty

$$\delta_\varphi \approx \frac{G \hbar}{\delta_r^2 c} , \quad (3)$$

where G is the Newton constant. This uncertainty relation between coordinates and gravitation field may be rewritten as

$$\left(\frac{\delta_\varphi}{c^2} \right) \left(\frac{\delta_r}{\lambda_P} \right)^2 \approx 1 , \quad (4)$$

with $\lambda_P = \sqrt{G\hbar/c^3}$ being the Planck length. Note that the uncertainty relation (3) disappears if at least one of the three conditions is satisfied: non-quantum case ($\hbar \rightarrow 0$); non-relativistic case ($c \rightarrow \infty$); absence of gravitation ($G \rightarrow 0$).

Let us discuss meaning of these relations. When gravity is present, the test-body of mass M generates, in addition to the Compton length (1), one more length scale, the Schwarzschild radius ⁵

$$\lambda_S = \frac{2GM}{c^2} . \quad (5)$$

At small M , $\lambda_S < \lambda_C$. But at $M > M_P/\sqrt{2}$, where

$$M_P = \sqrt{\frac{\hbar c}{G}} \quad (6)$$

is the Planck mass, the inequality changes its sign. If λ_S exceeds the proper dimension of the body, then it corresponds to the radius of the Schwarzschild horizon, which internal area is invisible for the frame, where the body is at rest. Therefore, it is λ_S that determines the position uncertainty for a heavier test-body. The minimal physical uncertainty is achieved when

$$\lambda_S = \lambda_C = \sqrt{2} \lambda_P . \quad (7)$$

Thus, if quantum theory describes gravitation by a field, the field should be non-local, since its coordinates can never be physically fixed with precision better than $\sqrt{2} \lambda_P$.

Furthermore, the combination φ/c^2 is just that appearing in the metric tensor ⁵. Therefore, Eq.(4) means that uncertainty of coordinates is related to uncertainties in the metric tensor and, may be, in other coordinates (and/or time). This may hint at non-commutativity of operators for different space-time coordinates in the future quantum gravity theory.

Such properties are not necessary for quantum theory of other known interactions in absence of gravity (though might work as well in some particular cases). This is related to a unique feature of gravitation: in contrast to any other interactions, there can not exist a test-body which would be neutral (sterile) in respect to gravity.

Acknowledgments

I thank I.T. Dyatlov for useful discussion. This work is supported by the Russian Science Foundation (Grant No.14-22-00281).

References

1. Jerzy Lukierski, arXiv:1404.6797.
2. A. Ashtekar, M. Reuter, C. Rovelli, arXiv:1408.4336.

3. B. Schulz, arXiv:1409.7977.
4. S. S. Schweber, H. A. Bethe, and F. de Hoffmann, *Mesons and Fields*, v.1, *Fields*, Row, Peterson, and Co., N.Y., 1955.
5. L. D. Landau, E. M. Lifshitz, *Course of Theoretical Physics*, v.2, *The Classical Theory of Fields*, Pergamon Press, 1971.